*IOSR Journal of Research & Method in Education (IOSR-JRME)*
*e-ISSN: 2320–7388,p-ISSN: 2320–737X Volume 5, Issue 3 Ver. I (May - Jun. 2015), PP 29-34*
*www.iosrjournals.org*

# The Experience Of Under Graduate Students with Literal Symbols

Dr Maher Al Ghanem
*Jubail University College, Al Jubail- 31961*

***Abstract:*** *This study investigated the result of an survey (experiment) where we observed misuses of literal symbols by university/college students when they perform operation on integration & differentiation problems*
***Keywords:*** *Differentiation, Integration, substitution, literal symbols*

## I. Introduction

In the transition from arithmetic to algebra ,the concept of variables appear. Mathematically speaking a variable is usually represented by a literal symbol, a letter from alphabet,which could represent any numerical value or other algebraic objects [1] In algebra and calculus certain letters are associated with certain roles. When variables are in question, the letters that first come in mind are x , y & z .As it is known fact that Mathematics is a science focusing on symbols in certain sense. Comprehension of the symbols used in mathematics, assigning them with different meaning and the use of different symbols or letters are particularly important for understanding the universal and abstract language of mathematics. University students' understandings of literal symbols in calculus were investigated under the theoretical framework of concept image. Tall and Vinner in 1981 introduces the notions of concept definition and concept image and make a distinction between the two.

In 1908 Poincare [3]made it very clear " What is definition ? For the philosopher or the scientist , it is a definition which applies to all the objects to be defined and applies only to them ; it is that which satisfies the rules of logic. But in education it is not; it is one that can be understood by the pupils.

Tall and Vinnerdefine the concept definition as the form of words used to specify the concept. A formal concept definition is one accepted by the mathematical community at large. As Tall and Vinner [2] mentioned that we can use mathematical concept without knowing the formal definitions. To explain how this occurs , they define concept image as 'the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and process'

In the present work we emphasize on the difficulties in understanding the literal symbols in integration & differentiation exercises in calculus.

## II. Purpose

The purpose of this study is to reveal the difficulties that undergraduatestudents are facing in using the symbols, letter in solving the integration & differentiation problems

## III. Participants

The students of Diploma course studying Calculus l at JIC/JUC

## IV. Procedure

The research was conducted in calculus l course in which topics of derivative , indefinite and definite integrals , area and volume are being taught .During the Midterm Exam and Final Exam ,the following questions of application was given to the students

<u>Question # 1</u>A spherical balloon is being inflated. Find the rate at which its volume increases with respect to its radius r when it is 15 cm, where volume of the sphere$V = \frac{4}{3}\pi r^3$

<u>Question # 2</u> Evaluate $\int \ln(1+u)\, du$

<u>Question # 3</u>Evaluate $\int \frac{du}{u^2+3u-4}$

<u>Question # 4</u>Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^2}}\, dx$

## V. Findings

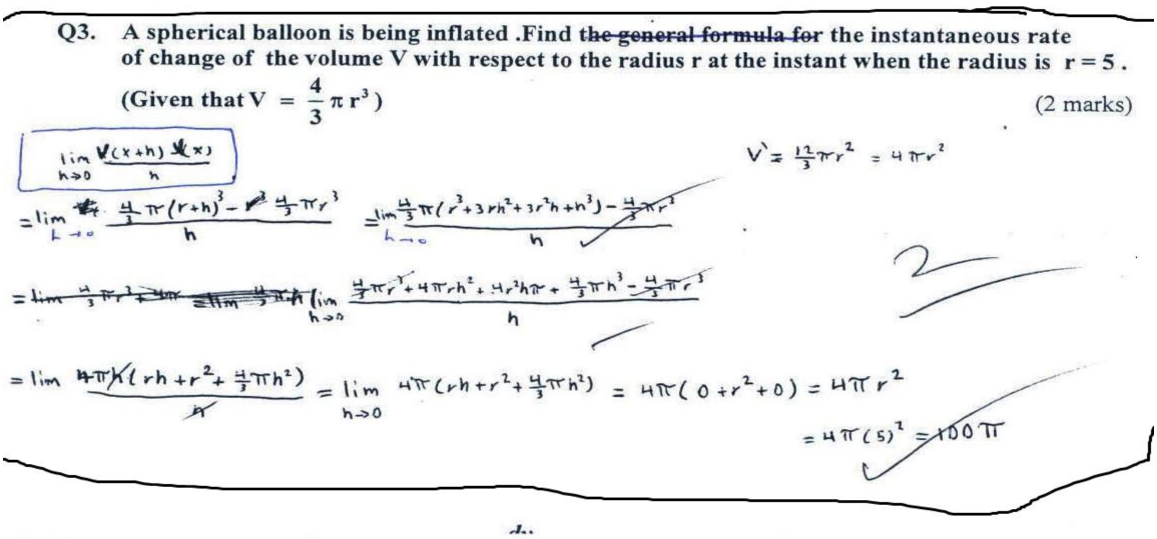
The quotations are prepared from the students' responses to the questions.
Figure 1 to 8 provide the samplesof the students' responses to the questions.
Table from 1 to4 provide the frequencies of students' responses to the questions.

Concerning literal expressions and symbols x , y , z , t are the letters frequently used in text books contents and lectures of Instructors . However besides the above listed letters , the present study used the letters u ,v , r which are as well relevant to the subject .

Whenever any mathematical problem in question , what comes first in mind of the students ass a literal expression is x . For them x is a conceptual image for a variable . In questions , besides the literal symbol x which the students are frequently using , there are also other letters such as t , u .

During the lectures on integration the lecturer used x as a literal symbol in questions . In this study the researchers attempted to measure the communication of students with other letters .Among the question posed to the students , the most striking one is about the integration by parts, the method of integration by parts uses two different variable substitutions ( two different literal expressions) for the given integral, the students had difficulties in performing the operation on literal variables and assigning another variable to the variable provided in the question.

**Figure#1**



**Q3.** A spherical balloon is being inflated .Find ~~the general formula for~~ the instantaneous rate of change of the volume V with respect to the radius r at the instant when the radius is $r = 5$ . (Given that $V = \frac{4}{3}\pi r^3$ ) (2 marks)

$\boxed{\lim_{h\to 0} \frac{V(x+h) - V(x)}{h}}$ $V' = \frac{12}{3}\pi r^2 = 4\pi r^2$

$= \lim_{h\to 0} \frac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h}$ $= \lim_{h\to 0} \frac{\frac{4}{3}\pi(r^3+3rh^2+3r^2h+h^3) - \frac{4}{3}\pi r^3}{h}$

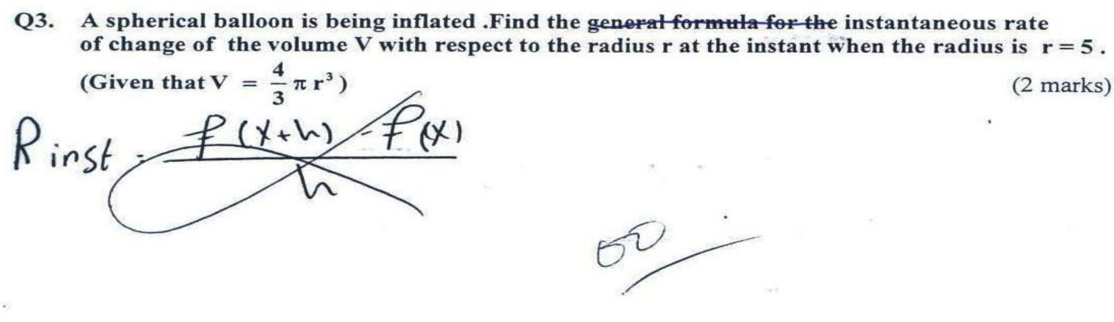
$= \lim_{h\to 0} \frac{\frac{4}{3}\pi r^3 + 4\pi r h^2 + 4r^2h\pi + \frac{4}{3}\pi h^3 - \frac{4}{3}\pi r^3}{h}$

$= \lim \frac{4\pi \cancel{h}(rh + r^2 + \frac{4}{3}\pi h^2)}{\cancel{h}} = \lim_{h\to 0} 4\pi(rh + r^2 + \frac{4}{3}\pi h^2) = 4\pi(0 + r^2 + 0) = 4\pi r^2$

$= 4\pi(5)^2 = 100\pi$

2

**Figure#2**

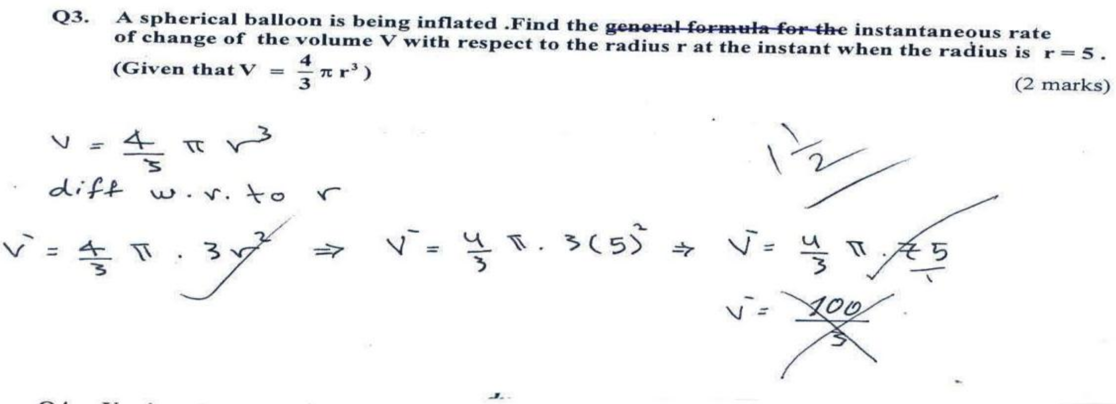


**Q3.** A spherical balloon is being inflated .Find the ~~general formula for the~~ instantaneous rate of change of the volume V with respect to the radius r at the instant when the radius is $r = 5$ . (Given that $V = \frac{4}{3}\pi r^3$ ) (2 marks)

$R_{inst} = \frac{f(x+h) - f(x)}{h}$

0

**Figure#3**



**Q3.** A spherical balloon is being inflated .Find the ~~general formula for the~~ instantaneous rate of change of the volume V with respect to the radius r at the instant when the radius is $r = 5$ . (Given that $V = \frac{4}{3}\pi r^3$ ) (2 marks)

$V = \frac{4}{3}\pi r^3$

diff w.r. to r

$V' = \frac{4}{3}\pi \cdot 3r^2 \Rightarrow V' = \frac{4}{3}\pi \cdot 3(5)^2 \Rightarrow V' = \frac{4}{3}\pi \cdot 75$

$V' = \frac{100}{3}$

$1\frac{1}{2}$

**Figure#4**

$$\int \ln(1+u)\,du = ?$$

$$\int u\,dv = uv - \int v\,du$$

$$u = \ln(1+u) \qquad dv = 1$$

$$du = \frac{1}{1+u}\,du \qquad v = v$$

$$\int \ln(1+u)\,du = \int \ln(1+u)\cdot v - \int \frac{1}{1+u}\,v\,du$$

1/2

**Figure#5**

$$\int \ln(1+u)\,du \qquad u = x$$

if $u = x \rightarrow \int \underset{dv}{1}.\underset{u}{\ln(1+x)}\,dx = ?$

$$u = \ln(1+x), \quad dv = 1$$

$$du = \frac{1}{1+x}\,dx \qquad \cancel{v} = x$$

3

$$\cancel{\int \ln(1+x)\,dx} \quad \int u\,dv = uv - \int v\,du$$

$$\int \underset{dv}{1}.\underset{u}{\ln(1+x)}\,dx = x\ln(1+x) - \int \frac{x}{1+x}\,dx$$

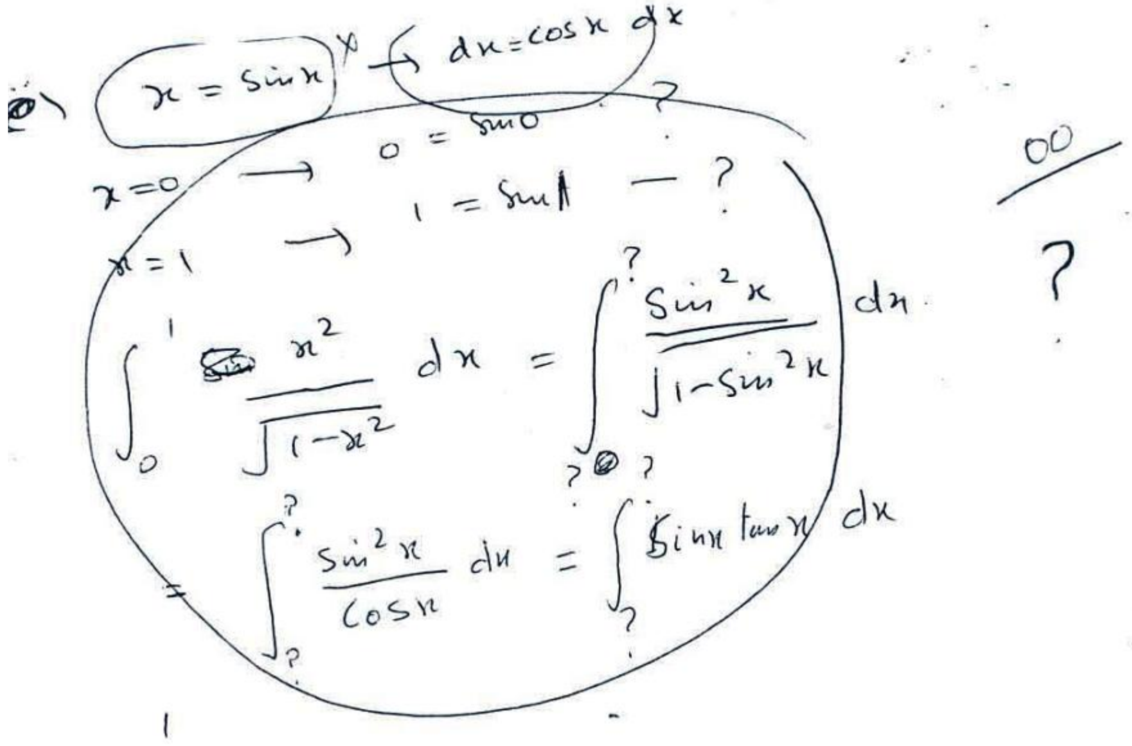
$$= x\ln(1+x) - \int \frac{x+1-1}{1+x}\,dx$$

$$= x\ln(1+x) - \int \left(1 - \frac{1}{1+x}\right)dx$$

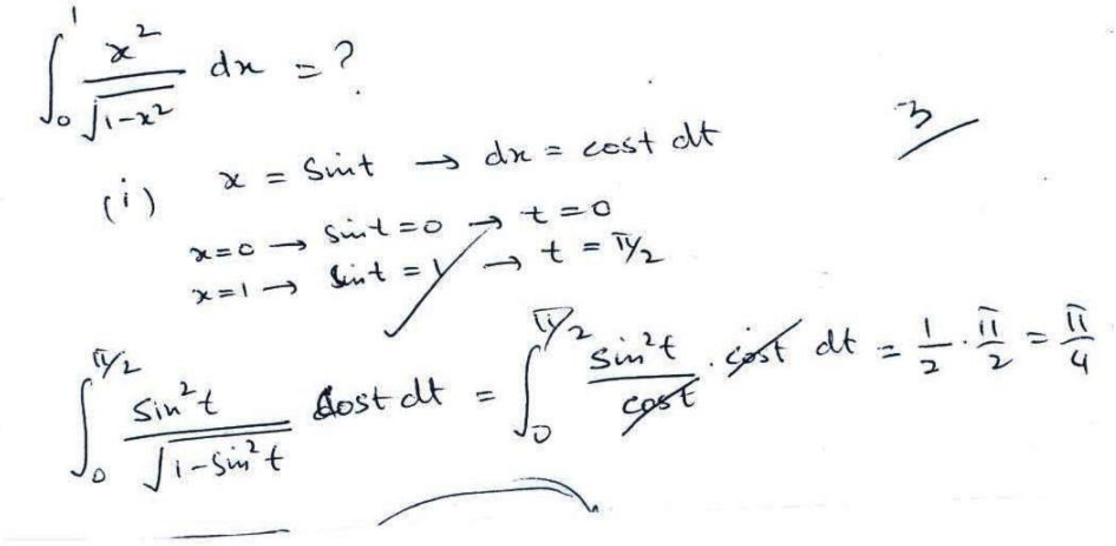
$$\int \ln(1+x)\,dx = x\ln(1+x) - x + \ln(1+x) + C$$

$$x = u \qquad \int \ln(1+u)\,du = u\ln(1+u) - u + \ln(1+u) + C$$

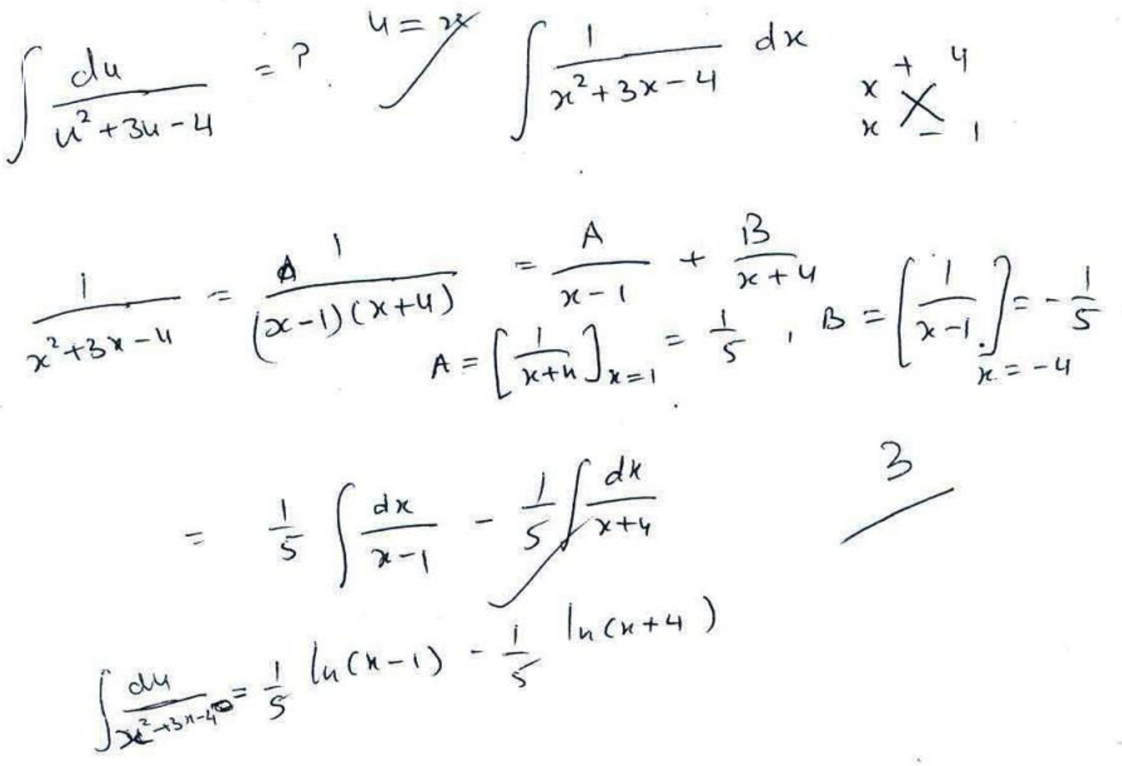
**Figure#6**




**Figure#7**



**Figure#8**



**Table# 1**

Students responses to question #1 in the midterm exam (N=100 )

Error due to the presentation of a literal expression in a different from i.e. r instead of x-- 55
Attempted correctly -- 25
No response --20

**Table# 2**

Students responses to question # 2 in final exam (N=90)

Attempting the solution by substituting the symbol given in the integral with symbol x and then using the formula $\int udv = uv - \int vdu$ with which they are familiar --52

Using the formula $\int udv = uv - \int vdu$ without changing the variables --24
No response --14

**Table# 3**

Students responses to question # 3 in final exam (N= 90)

Doing the operation of partial fraction immediately transferring the literal symbol u into x -- 56
Doing the operation of partial fraction with the same given symbol --25
No response --9

**Table# 4**

Students responses to question # 4 in final exam (N= 90)

Attempting the solution by substituting x = sin t --47
Difficulty experience in transferring between literal expression as wrongly substitution x = sin x --23
No response --20

**1. Observations& Conclusion**

<u>For question #1</u>

i 70% of 700 Students were not able to differentiate it with respect to r as they are familiar with x
ii If x has been chosen in place of r , the student would have done much better

**For question #2**

It is for Integration by parts where they find the difficulty in applying formula $\int udv = uv - \int vdu$ as the symbol coincides with the variable u which is given in the figure # 4

**For question #3**

It is for the integration by using partial fraction where the students are able to solve it by changing the variable from u to x otherwise the students are mixed with the their answers

**For question # 4**

It is for the integration by using trigonometric substitution where some students are facing difficulties in choosing the correct symbol in substitutions (figure # 6)

In short, we can conclude that the students should be familiarizes with different type of symbols from the very begining, teacher should be train to deliver the lectures accordingly; however, the textbooks should include problems with different symbols.

## References

[1]. Christou KP, Vosniadou S; How students interpret literal symbols in Algebra: A conceptual change approach, 17th annual conference ofthe cognitive science society,Italy, 453-458, (2005).

[2]. Tall DO, VinnerS: Concept image and concept definition in mathematics with particular reference to limit and continuity. Edu.Stu.Math,12: 151-169 (1981).

[3]. Poincare H. 1908: ‘Science et Method’ (translated by Francis Maitland 1913) English Version Dover Publications.